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TEMPERATURE RESPONSE OF AN INFINITE FLAT PLATE WITH UNSYMMETRICAL BOUNDARY CONDITIONS

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ARO, Inc.

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NOTICES

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FOREWORD

The work reported herein was done at the request of Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Station, Tennessee, under Program Element 65402234.

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This technical report has been reviewed and is approved.

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ABSTRACT

Exact solutions for the transient temperature distribution and the stored energy in an infinite plate of finite thickness are presented for the case of different convective environments at each face of the plate. The solution is general and contains numerous limiting cases, including that of steady state. Eigenvalues are given for many combinations of the system Biot numbers for the initial response period. An example is presented to illustrate the application of the solution to the practical problem of a rocket engine diffuser.

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ILLUSTRATIONS

TABLE

NOMENCLATURE

H	Parameter, h/k, 1/ft
h	Surface heat-transfer coefficient, Btu/hr-ft^2-$^\circ$F
K	Thermal diffusivity of plate, ft^2/hr
k	Thermal conductivity of plate, Btu/hr-ft-$^\circ$F
ℓ	Thickness of plate, ft

N_B	Biot number, $h\ell/k$
Q	Energy stored in plate in time t, Btu/ft^2
t	Time, hr
V	Initial plate temperature, ^oF
v	Variable plate temperature, ^oF
w	Function of temperature
x	Distance from left face of plate, ft
z	Transient function of temperature
α	Ratio of Biot numbers, $N_{B_1}/N_{B_2} = h_1/h_2$
β	Parameter, 1/ft (constant of separation)
δ	Temperature ratio, $(V-v_2)/(v_2-v_1)$
ϵ	Parameter, $\beta\ell$
θ	Temperature ratio, $(V-v_1)/(v_2-v_1)$
ψ	Temperature ratio, $(V-v_1)/(v_2-v_1)$

SUBSCRIPTS

o	Refers to a maximum condition
1	Refers to face at x = 0
2	Refers to face at $x = \ell$

SECTION I
INTRODUCTION

The transient response of an infinite flat plate of finite thickness has been analyzed for many cases (Refs. 1 through 5). However, the most general solution for convective environments (different surface heat-transfer coefficients and thermal environments on each side of the plate) is not available, although the possibility of the solution is mentioned in Ref. 1. The transient response of a plate subjected to unsymmetrical boundary conditions is very important in many analyses. For example, the transient time is the prime period of interest in evaluating the behavior and application of structures subjected to unsymmetrical boundary conditions, such as exhaust gas diffusers for simulating the high altitude environment of rocket engines, rocket engine nozzles, ejectors, tunnel walls of high temperature short run time test facilities, nozzles of intermittently operated rockets, and components of aircraft and missiles in high speed flight. In many of the above cases, the engineer is ultimately interested in predicting coolant flow rates necessary to keep the wall within structural and material temperature limits. It is reasonable to expect that the coolant rate necessary for a short time test or exposure may be of a reasonable magnitude, whereas the coolant rate necessary for steady-state operation may be completely unreasonable in some of the above applications.

This analysis presents an exact solution for the temperature response in a solid bounded by two parallel planes with unsymmetrical boundary conditions. Implicit in the solution is the capability of predicting a coolant flow rate necessary to keep an exposed wall within structural and temperature limits.

SECTION II
ANALYSIS

2.1 PHYSICAL SYSTEM

The physical system considered in this analysis is shown in Fig. 1. An infinite plate of finite thickness ℓ is initially at a uniform temperature $f(x)$ throughout. At time $t \geq o$, the face at $x = o$ is exposed to a high temperature convective environment at temperature v_1. Similarly, for time $t \geq o$, the face at $x = \ell$ is exposed to a lower temperature convective environment at temperature v_2. Assume that the surface heat-transfer coefficients h_1 and h_2 are uniform and constant at $x = o$ and $x = \ell$, respectively. The thermal conductivity and thermal diffusivity are given by k and K, respectively, and are assumed to be independent of temperature and position.

2.2 MATHEMATICAL MODEL

A basic energy balance on the plate shows that the partial differential equation describing the temperature distribution in the plate is given by

$$\frac{\partial v}{\partial t} = K \frac{\partial^2 v}{\partial x^2} \qquad (1)$$

This equation is subject to the following boundary conditions:

$$k \frac{\partial v}{\partial x} - h_1(v - v_1) = 0 \qquad \text{at } x = 0$$

$$k \frac{\partial v}{\partial x} + h_2(v - v_2) = 0 \qquad \text{at } x = \ell$$

and the initial condition:

$$v = f(x) \qquad \text{at } t \leq 0$$

The above system of equations can be solved by many different techniques. However, the principle of superposition is especially convenient for this problem. Assuming that the solution can be expressed as

$$v(x,t) = u(x) + w(x,t) \qquad (2)$$

where $u(x)$ is the steady-state contribution to temperature and $w(x,t)$ is the transient contribution, then $u(x)$ must satisfy the differential equation

$$\frac{d^2 u}{dx^2} = 0 \qquad 0 \leq x \leq \ell \qquad (3)$$

subject to the following boundary conditions:

$$k \frac{du}{dx} - h_1(u - v_1) = 0 \qquad \text{at } x = 0$$

$$k \frac{du}{dx} + h_2(u - v_2) = 0 \qquad \text{at } x = \ell$$

The function $w(x,t)$ must then satisfy the partial differential equation

$$\frac{\partial w}{\partial t} = K \frac{\partial^2 w}{\partial x^2} \qquad 0 \leq x \leq \ell \qquad (4)$$

subject to the following boundary and initial conditions

$$k\frac{\partial w}{\partial x} - h_1 w = 0 \quad \text{at } x = 0$$

$$k\frac{\partial w}{\partial x} + h_2 w = 0 \quad \text{at } x = \ell$$

$$w = f(x) - u \quad \text{at } t \leq 0$$

It may be readily shown that the solution to the system of Eq. (3) is

$$u = \frac{H_1 H_2 (v_2 - v_1)x + H_1 v_1 (1 + H_2 \ell) + H_2 v_2}{H_1 + H_2(1 + H_1 \ell)} \tag{5}$$

where $H_1 \equiv \frac{h_1}{k}$ and $H_2 \equiv \frac{h_2}{k}$

It may be shown that by using the product-type solution, the solution to the system of Eq. (4) is

$$w = \sum_{n=1}^{\infty} Z_n(x)\, e^{-K\beta_n^2 t} \int_0^{\ell} Z_n(x') \left[f(x') - u(x')\right] dx' \tag{6}$$

where

$$Z_n(x) = -\frac{\left[2(\beta_n^2 + H_2^2)\right]^{\frac{1}{2}} \left[\beta_n \cos(\beta_n x) + H_1 \sin(\beta_n x)\right]}{\left\{(\beta_n^2 + H_1^2)\left[\ell(\beta_n^2 + H_2^2) + H_2\right] + H_1(\beta_n^2 + H_2^2)\right\}^{\frac{1}{2}}} \tag{7}$$

where β_n are the positive roots of

$$(\beta_n^2 - H_1 H_2) \sin(\beta_n \ell) = \beta_n(H_1 + H_2) \cos(\beta_n \ell) \tag{8}$$

Therefore, the solution to Eq. (1), using the assumption of Eq. (2), is the sum of Eqs. (5) and (6), or

$$v(x,t) = \frac{H_1 H_2 (v_2 - v_1)\, x + H_1 v_1 (1 + H_2 \ell) + H_2 v_2}{H_1 + H_2 (1 + H_1 \ell)} \tag{2}$$

$$+ \sum_{n=1}^{\infty} Z_n(x) e^{-K\beta_n^2 t} \int_0^{\ell} Z_n(x') \left[f(x') - u(x')\right] dx'$$

It is beyond the scope of this work to prove that Eq. (2) represents the unique solution to the system of Eq. (1) and that Eq. (2) is a uniformly convergent solution; uniqueness and uniform convergence may be shown readily.

For simplicity, let the general solution (Eq. [2]) be modified by assuming that

$$f(x) = f\,(x') = V = \text{initial plate temperature} \tag{9}$$

Substitute Eq. (9) into Eq. (2) and integrate to obtain

$$v - v_1 = \frac{(v_2 - v_1)\left[H_2 + H_1 H_2 x\right]}{H_1 + H_2(1 + H_1 \ell)}$$

$$+ 2 \sum_{n=1}^{\infty} \frac{(\beta_n^2 + H_2^2)\left[\beta_n \cos(\beta_n x) + H_1 \sin(\beta_n x)\right]}{(\beta_n^2 + H_1^2)\left[\ell(\beta_n^2 + H_2^2) + H_2\right] + H_1(\beta_n^2 + H_2^2)} \left[\left\{\frac{H_1(V - v_1) + (H_2 + H_1 H_2 \ell)(V - v_2)}{H_1 + H_2(1 + H_1 \ell)}\right.\right.$$

$$\left. - \frac{H_1^2 H_2 (v_2 - v_1)}{\beta_n^2 \left[H_1 + H_2(1 + H_1 \ell)\right]}\right\} \sin(\beta_n \ell) + \frac{H_1}{\beta_n}\left\{\frac{H_1 H_2 \ell(v_2 - V) + (H_1 + H_2)(v_1 - V)}{H_1 + H_2(1 + H_1 \ell)}\right\} \cos(\beta_n \ell)$$

$$\left. + \frac{H_1}{\beta_n}(V - v_1)\right] e^{-\beta_n t} \tag{10}$$

Equation (10) will be more convenient to work with in a dimensionless form. Using the dimensionless temperature ratios θ, δ, ψ, N_{B_1} based on h_1, N_{B_2} based on h_2, and the dimensionless parameter α and ϵ_n, Eq. (10) may be written

$$\theta = \frac{1}{1 + \alpha + N_{B_1}} \left\{1 + N_{B_1}(x/\ell)\right\}$$

$$+ 2 \sum_{n=1}^{\infty} \left(\frac{N_{B_1}}{\epsilon_n}\right) \frac{\left\{1 + \left(\frac{N_{B_2}}{\epsilon_n}\right)^2\right\}\left\{\cos\left(\frac{\epsilon_n x}{\ell}\right) + \frac{N_{B_1}}{\epsilon_n} \sin\left(\frac{\epsilon_n x}{\ell}\right)\right\}}{\left\{1 + \left(\frac{N_{B_1}}{\epsilon_n}\right)^2\right\}\left\{\epsilon_n\left[1 + \left(\frac{N_{B_2}}{\epsilon_n}\right)^2\right] + \frac{N_{B_2}}{\epsilon_n}\right\} + \frac{N_{B_1}}{\epsilon_n}\left[1 + \left(\frac{N_{B_2}}{\epsilon_n}\right)^2\right]}$$

$$\left[\left\{\frac{\epsilon_n}{N_{B_1}}\left[\alpha\,\psi + (1 + N_{B_1})\delta\right] - \frac{N_{B_1}}{\epsilon_n}\right\} \sin(\epsilon_n) - \left\{N_{B_1}\,\delta + (1 + \alpha)\,\psi\right\} \cos(\epsilon_n) + \psi\,(1 + \alpha + N_{B_1})\right]$$

$$e^{-\epsilon_n^2 \frac{Kt}{\ell^2}} \tag{11}$$

The equation for the eigenvalues, Eq. (8) in dimensionless form becomes

$$\tan(\epsilon_n) = \frac{\epsilon_n(N_{B_1} + N_{B_2})}{\epsilon_n^2 - N_{B_1}N_{B_2}} \tag{12}$$

Equations (11) and (12) are sufficient to determine the dimensionless temperature distribution in an infinite plate of finite thickness exposed to unsymmetrical boundary conditions.

In addition to checking Eq. (11) for uniqueness and uniform convergence, one may also show that it reduces properly to various "special cases". The Heisler or Groeber-type solution (face at $x = 0$ insulated or $N_{B_1} = 0$, and N_{B_2} finite) available in most textbooks on heat transfer is readily obtained by letting $N_{B_1} = 0$ in Eq. (11). Other cases, such as both faces insulated ($N_{B_1} = N_{B_2} = 0$), zero thermal resistance at $x = 0$ ($N_{B_1} = \infty$), and N_{B_2} finite (or vice versa), and zero thermal resistance at both faces ($N_{B_1} = N_{B_2} = \infty$), are all readily obtained by proper reduction of Eq. (11). For the steady-state case, Eq. (11) reduces to

$$\theta(x) = \frac{1 + N_{B_1}(x/\ell)}{1 + \alpha + N_{B_1}} \tag{13}$$

The total energy stored in the plate per unit area Q, in time t, is given by

$$Q = k(v_2 - v_1)\int_0^t \left\{\frac{\partial\theta}{\partial t}\bigg|_{x=\ell} - \frac{\partial\theta}{\partial x}\bigg|_{x=0}\right\} dt \tag{14}$$

The maximum energy stored in the plate per unit area is defined as

$$Q_o \equiv \rho c \ell (v_2 - v_1) \tag{15}$$

Substituting Eq. (11) into Eq. (14), dividing by Eq. (15), and performing the indicated operations gives the ratio of the total heat flow into or out of the plate in time t, to the maximum energy of the plate, as

$$\frac{Q}{Q_o} = \frac{2N_{B_1}}{1+\alpha+N_{B_1}} \sum_{n=1}^{\infty} \left(\frac{1}{\epsilon_n^2}\right) \frac{\left\{1 + \left(\frac{N_{B_2}}{\epsilon_n}\right)^2\right\}\left\{\frac{N_{B_1}}{\epsilon_n}\left[\cos(\epsilon_n) - 1\right] - \sin(\epsilon_n)\right\}}{\left\{1 + \left(\frac{N_{B_1}}{\epsilon_n}\right)^2\right\}\left\{\epsilon_n\left[1 + \left(\frac{N_{B_2}}{\epsilon_n}\right)^2\right] + \frac{N_{B_2}}{\epsilon_n}\right\} + \frac{N_{B_1}}{\epsilon_n}\left[1 + \left(\frac{N_{B_2}}{\epsilon_n}\right)^2\right]}$$

$$\left[\left\{\frac{\epsilon_n}{N_{B_1}}\left[\alpha\psi + (1 + N_{B_1})\delta\right] - \frac{N_{B_1}}{\epsilon_n}\right\}\sin(\epsilon_n) - \left\{N_{B_1}\delta + (1+\alpha)\psi\right\}\cos(\epsilon_n) + \psi(1+\alpha+N_{B_1})\right]$$

$$\left(1 - e^{-\epsilon_n^2 \frac{Kt}{\ell^2}}\right) \tag{16}$$

Equations (11), (12), and (16) are sufficient to determine the temperature-time history and the total heat flow into or out of the plate as functions of time, the system Biot numbers, the environment temperature at the faces of the plate, and the initial temperature of the plate.

SECTION III
RESULTS

3.1 GENERAL

The eigenvalues ϵ_n were calculated from Eq. (12) by computer for a wide range of characteristic Biot numbers N_{B_1} and N_{B_2}. The first ten positive roots were calculated for each combination of N_{B_1} and N_{B_2}, each root being accurate to four places. Table I lists values of ϵ_n for all possible combinations of the following:

N_{B_1} = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 1.0, 2.0, 4.0, 6.0, 8.0, 10.0, 20.0, 100.0, ∞.

N_{B_2} = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 1.0, 2.0, 4.0, 6.0, 8.0, 10.0, 20.0, 100.0, ∞.

The series in Eq. (11) and (16) converge rapidly with ten or fewer roots of Eq. (12) for all values of $Kt/\ell^2 \geq 0.01$.

Unfortunately, in Eq. (11) it is not possible to obtain only the system temperature terms on the left side of the equation and only the Biot and Fourier numbers on the right side of the equation. This eliminates the possibility of a general dimensionless plot of Eq. (11). However, if it is assumed that the initial temperature of the plate V equals the environment temperature v_2, then the dimensionless temperature parameters δ and ψ become 0 and 1, respectively. The right side of Eq. (11) then becomes a function of N_{B_1}, N_{B_2}, and Kt/ℓ^2 only. This is a reasonable assumption for some of the typical applications that were mentioned. With this assumption, the quantitative effect of varying conditions of surface heat transfer, plate thickness, run time, and material properties on the temperature response of the plate can be determined. It should be emphasized that the application of these equations is not restricted to any one material, environmental temperature difference, heat-transfer coefficient, or run time because of the dimensionless character of the solution.

3.2 ILLUSTRATIVE EXAMPLE

The solution can best be appreciated by considering a typical problem. Consider the case of an exhaust gas diffuser for simulating the high altitudes necessary to evaluate the performance of rocket engines.

It is desired to know the temperature response of the diffuser wall because the response must be limited to maintain the structural integrity of the diffuser. The wall is initially at temperature V, cooled by a constant temperature $v_2 = V$ water reservoir at $x = \ell$, and at $t > 0$ is subjected to a high temperature gas flow at temperature v_1 at $x = 0$. For this example let N_{B_1} be 0.3 and N_{B_2} 0.6, the wall be of 3/8-in. thick mild steel, the cooling water temperature be 70°F, and the hot gas temperature be 4,000°F. Determine the length of time for the wall at face $x = 0$ to reach 800°F, which will be assumed to be the limiting structural temperature.

$$\theta = v - v_1/v_2 - v_1 = 1260\text{-}4460/530\text{-}4460 = -3200/-3930 = 0.814$$

Referring to Fig. 2, which has example temperature response curves plotted for the face at $x = 0$, gives a Fourier number $Kt/\ell^2 = 0.4$ for $N_{B_1} = 0.3$ and $N_{B_2} = 0.6$. The thermal diffusivity K for mild steel is $\simeq 0.49$. Substituting gives

$$\frac{0.49\ t}{\left(\frac{3/8}{12}\right)^2} = 0.4$$

$$t = \frac{0.4\ (9.766 \times 10^{-4})}{0.49} = 7.972 \times 10^{-4} = 0.000797\ \text{Hr}$$

$$\text{or } t = 2.87 \text{ sec}$$

This is one example of the use of the curves. For a given run time, the plate temperature could have been determined just as readily. For other given conditions the plate thickness, coolant water temperature, or coolant side Biot number can be found. Figure 3 gives example surface temperature response curves for the face at $x = \ell$. From Figs. 2 and 3, it can be seen that the magnitude of N_{B_2} may be critical in reducing the surface temperature response of the wall. For example, consider the curve $N_{B_1} = N_{B_2} = 0.2$ in Fig. 2. For a given material wall thickness and run time, this curve represents the temperature response of a wall with equal heat-transfer coefficients at $x/\ell = 0$ and $x/\ell = 1$. If all conditions remain the same except that the heat-transfer coefficient at $x/\ell = 1.0$ is quadrupled, the new response curve has the Biot numbers $N_{B_1} = 0.2$ and $N_{B_2} = 0.8$. Depending on the magnitude of the Fourier number, the reduction in the temperature response of the wall may or may not be significant. For instance, if $Kt/\ell^2 = 1.0$ the difference in the response of the curves with Biot numbers $N_{B_1} = N_{B_2} = 0.2$ and $N_{B_1} = 0.2$, $N_{B_2} = 0.8$ is about four percent; however, at steady state the difference is about 35 percent. Figure 3 shows that the wall

temperature at $x = \ell$ responds at a slower rate than the wall temperature at $x = 0$, as one would expect. Also, the temperature response decreases with increasing N_{B_2} for a fixed N_{B_1}.

Figure 4 gives example heat storage curves for various combinations of N_{B_1} and N_{B_2} as a function of the Fourier number Kt/ℓ^2.

SECTION IV
CONCLUSIONS

Through the use of Eqs. (11), (12), and (16), general temperature-time plots and energy-stored plots can be developed to cover all cases of interest for a given situation. For design purposes the temperature distribution in a wall is of importance in determining thermal stresses, structural integrity, and peak surface temperatures. For the example considered, it is shown that the ratio of N_{B_2} to N_{B_1} can be of significant importance in reducing the temperature response of a wall. For a specific problem, the equations may be used to determine the most economical combination of wall material, wall thickness, and coolant flow rate and temperature; or even if it is feasible to limit a given wall to an acceptable temperature response.

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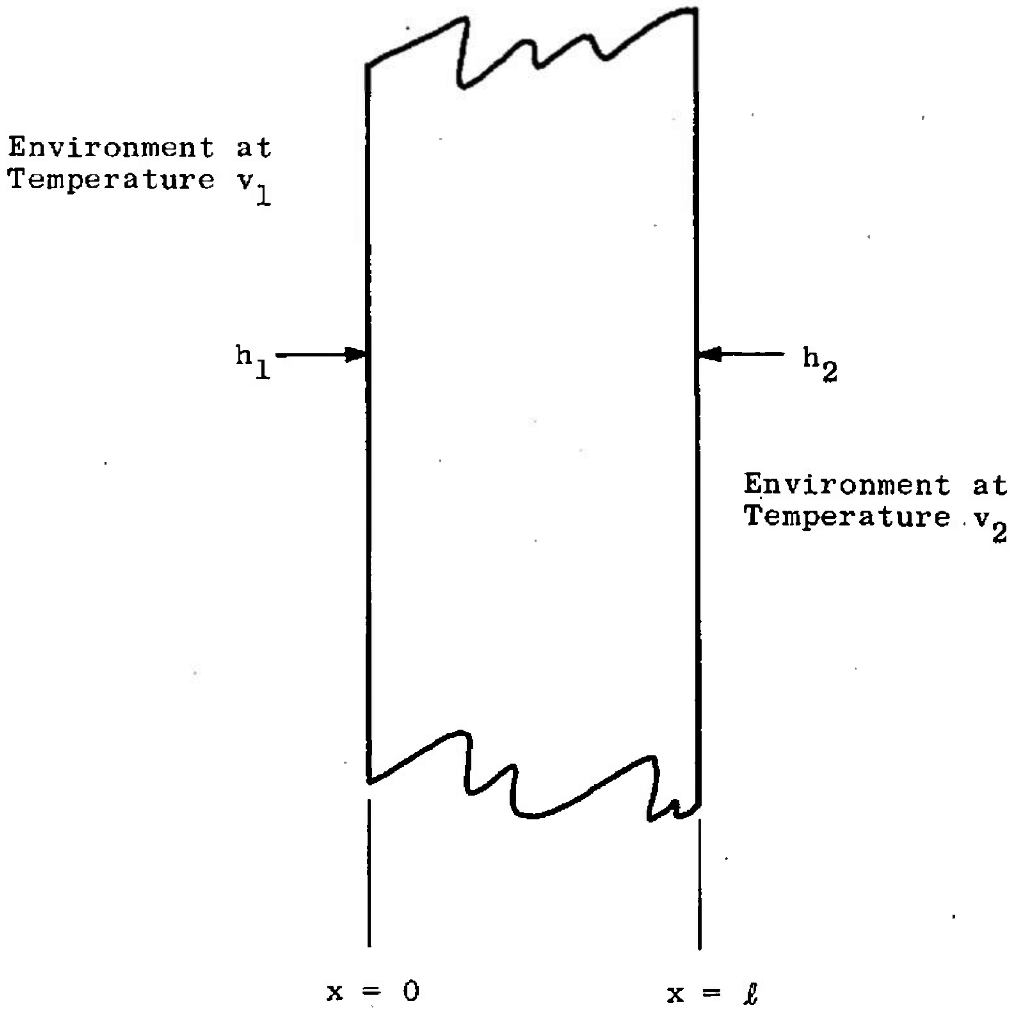

Fig. 1 Plate with Unsymmetrical Boundary Conditions

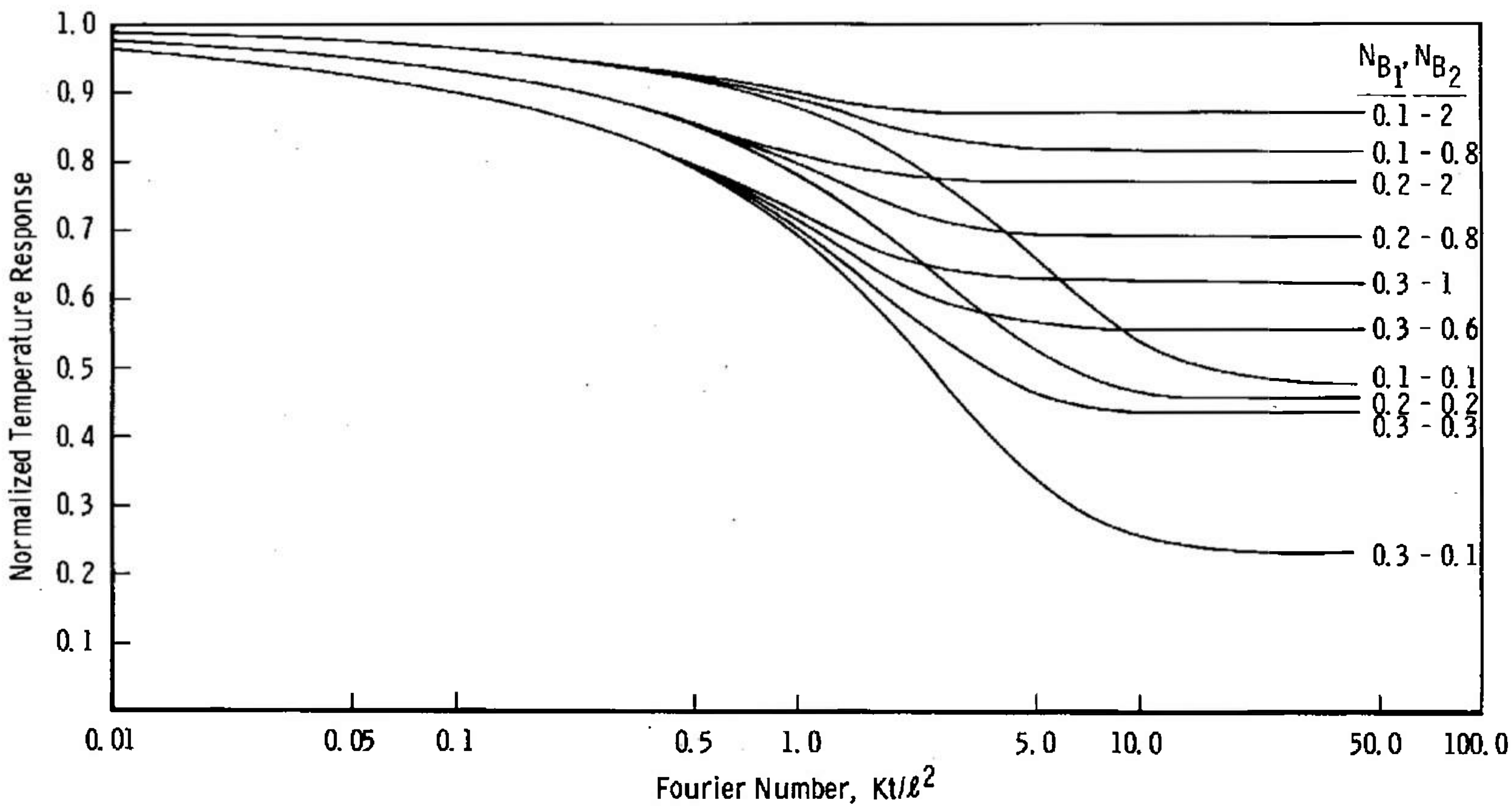

Fig. 2 Temperature Response for Plate at x = 0

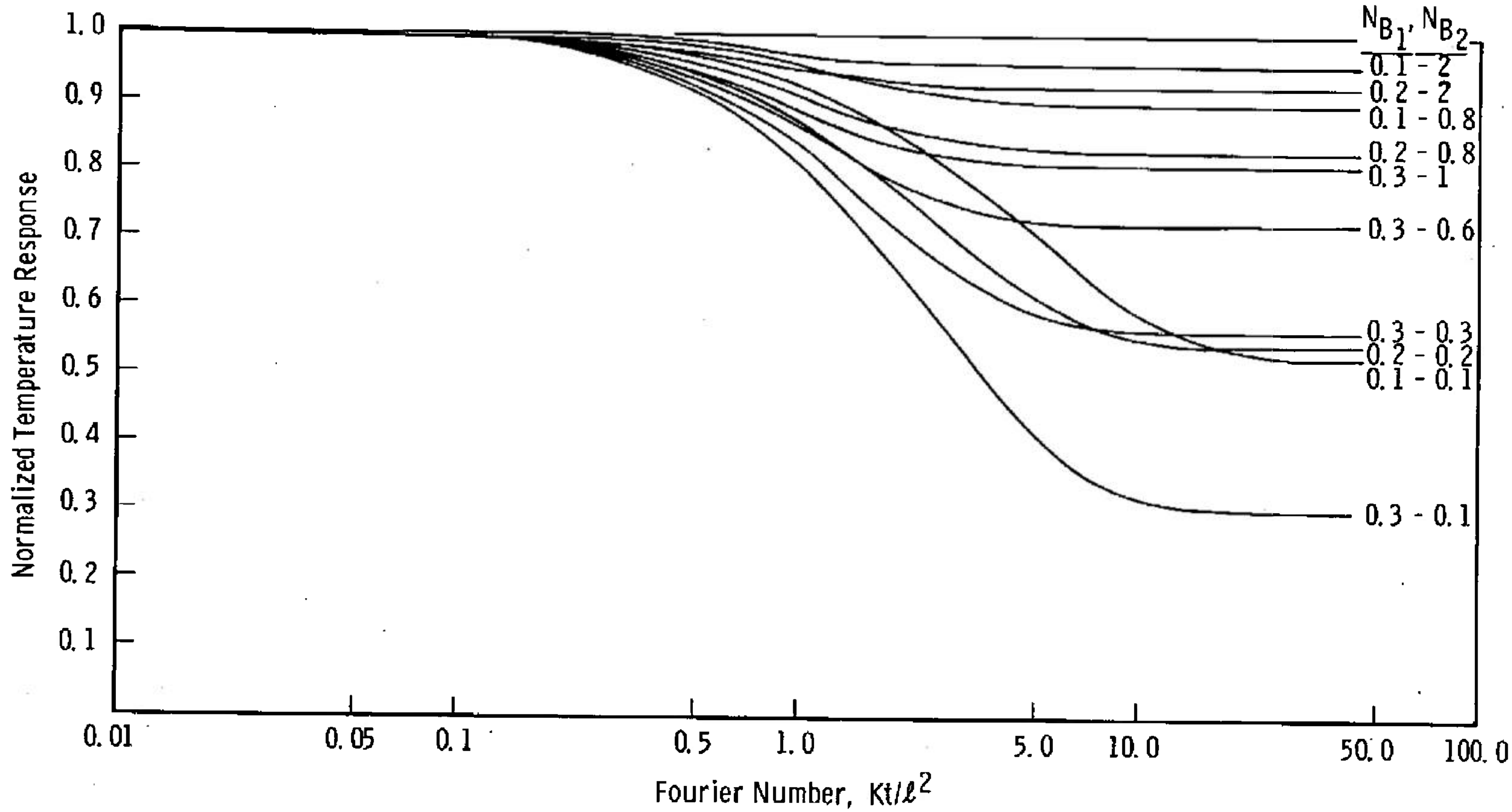

Fig. 3 Temperature Response for Plate at $x = \ell$

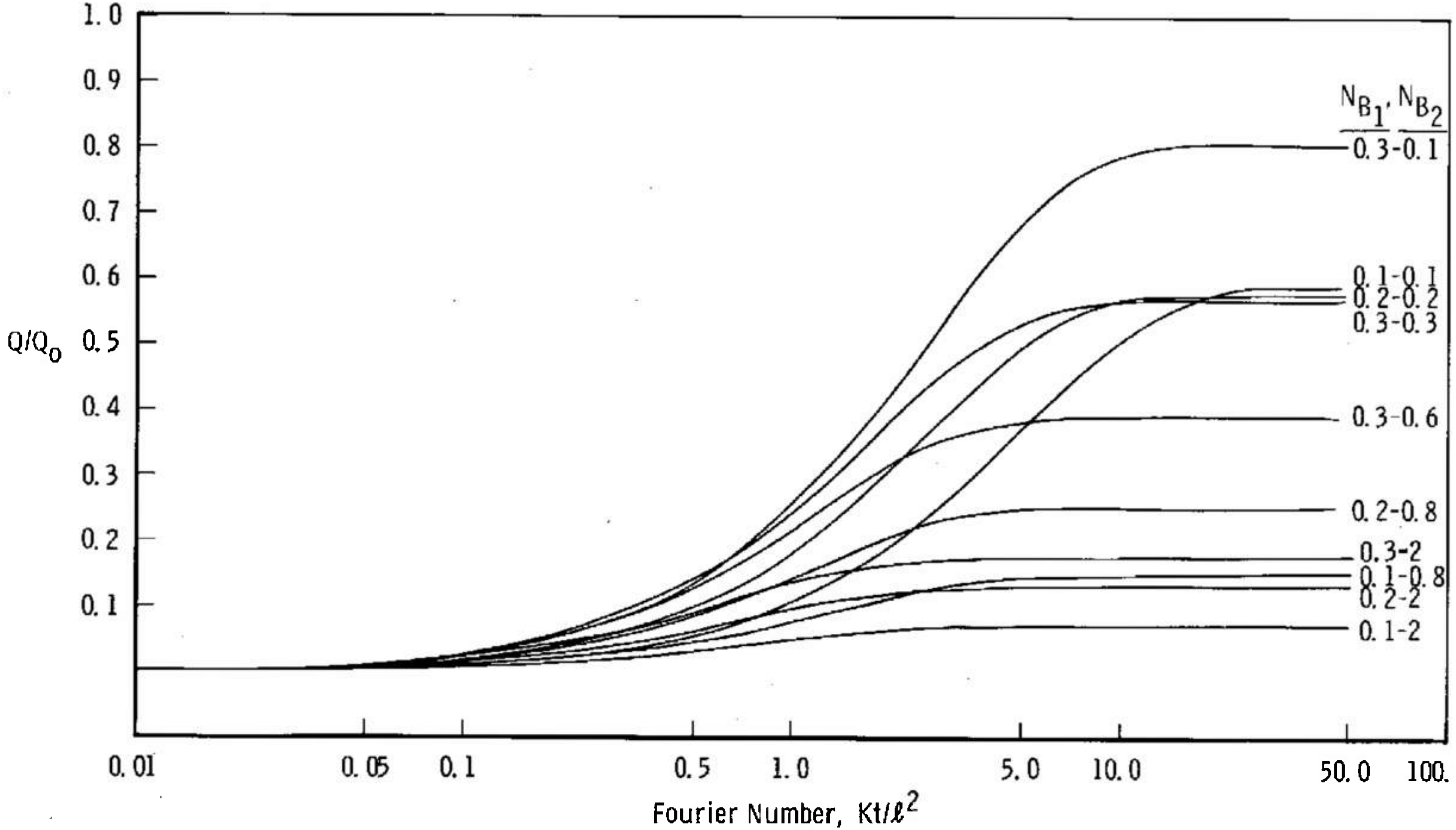

Fig. 4 Heat Storage in the Plate

TABLE I

FIRST TEN POSITIVE ROOTS OF THE TRANSCENDENTAL EQUATION $(\epsilon_n^2 - N_{B1} N_{B2}) \tan(\epsilon_n) = (N_{B1} + N_{B2}) \epsilon_n$ FOR VARIOUS BIOT NUMBER COMBINATIONS

N_{B1}	N_{B2}	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_9	ϵ_{10}
.0	.1	.3111	3.1731	6.2991	9.4354	12.5743	15.7143	18.8549	21.9957	25.1367	28.2779
.0	.2	.4328	3.2039	6.3148	9.4459	12.5823	15.7207	18.8602	22.0002	25.1407	28.2814
.0	.3	.5218	3.2341	6.3305	9.4565	12.5902	15.7270	18.8655	22.0048	25.1447	28.2849
.0	.4	.5932	3.2636	6.3461	9.4670	12.5981	15.7334	18.8707	22.0093	25.1486	28.2885
.0	.5	.6533	3.2923	6.3616	9.4775	12.6060	15.7397	18.8760	22.0139	25.1526	28.2920
.0	1.0	.8603	3.4256	6.4373	9.5293	12.6453	15.7713	18.9024	22.0365	25.1724	28.3096
.0	2.0	1.0769	3.6436	6.5783	9.6296	12.7223	15.8336	18.9547	22.0815	25.2119	28.3448
.0	4.0	1.2646	3.9352	6.8140	9.8119	12.8678	15.9536	19.0565	22.1697	25.2896	28.4142
.0	6.0	1.3496	4.1116	6.9924	9.9667	12.9988	16.0654	19.1531	22.2545	25.3650	28.4820
.0	8.0	1.3978	4.2264	7.1263	10.0949	13.1141	16.1675	19.2435	22.3351	25.4374	28.5476
.0	10.0	1.4289	4.3058	7.2281	10.2003	13.2142	16.2594	19.3270	22.4108	25.5064	28.6106
.0	20.0	1.4961	4.4915	7.4954	10.5117	13.5420	16.5864	19.6439	22.7131	25.7923	28.8600
.0	100.0	1.5552	4.6658	7.7764	10.8871	13.9981	17.1093	20.2208	23.3327	26.4450	29.5577
.0	INFY	1.5708	4.7124	7.8540	10.9956	14.1372	17.2788	20.4204	23.5619	26.7035	29.8451
.1	.1	.4435	3.2040	6.3149	9.4460	12.5823	15.7207	18.8602	22.0002	25.1407	28.2814
.1	.2	.5389	3.2343	6.3306	9.4565	12.5902	15.7270	18.8655	22.0048	25.1447	28.2849
.1	.3	.6150	3.2639	6.3462	9.4670	12.5981	15.7334	18.8708	22.0093	25.1486	28.2885
.1	.4	.6788	3.2928	6.3617	9.4775	12.6060	15.7397	18.8760	22.0139	25.1526	28.2920
.1	.5	.7357	3.3211	6.3771	9.4880	12.6139	15.7461	18.8813	22.0184	25.1566	28.2955
.1	1.0	.9293	3.4525	6.4524	9.5397	12.6531	15.7776	18.9077	22.0410	25.1764	28.3132
.1	2.0	1.1402	3.6680	6.5929	9.6397	12.7301	15.8399	18.9599	22.0860	25.2159	28.3483
.1	4.0	1.3260	3.9576	6.8278	9.8217	12.8754	15.9598	19.0616	22.1741	25.2935	28.4177
.1	6.0	1.4107	4.1333	7.0057	9.9763	13.0063	16.0715	19.1583	22.2589	25.3689	28.4854
.1	8.0	1.4589	4.2478	7.1394	10.1044	13.1215	16.1735	19.2486	22.3395	25.4413	28.5510
.1	10.0	1.4899	4.3271	7.2411	10.2096	13.2215	16.2653	19.3321	22.4152	25.5103	28.6140
.1	20.0	1.5572	4.5126	7.5082	10.5208	13.5491	16.5922	19.6489	22.7174	25.7961	28.8834
.1	100.0	1.6164	4.6869	7.7891	10.8962	14.0052	17.1151	20.2257	23.3370	26.4488	29.5611
.1	INFY	1.6320	4.7335	7.8667	11.0047	14.1442	17.2845	20.4252	23.5662	26.7073	29.8485
.2	.2	.6221	3.2640	6.3462	9.4670	12.5981	15.7334	18.8708	22.0093	25.1486	28.2885
.2	.3	.6912	3.2931	6.3617	9.4775	12.6060	15.7397	18.8760	22.0139	25.1526	28.2920
.2	.4	.7503	3.3216	6.3772	9.4880	12.6139	15.7461	18.8813	22.0184	25.1566	28.2955
.2	.5	.8019	3.3494	6.3925	9.4984	12.6218	15.7524	18.8866	22.0229	25.1606	28.2991
.2	1.0	.9899	3.4789	6.4675	9.5500	12.6610	15.7839	18.9130	22.0455	25.1804	28.3167

TABLE I (Continued)

N_{B1}	N_{B2}	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_9	ϵ_{10}
.2	2.0	1.1970	3.6921	6.6074	9.6499	12.7378	15.8461	18.9652	22.0905	25.2198	28.3518
.2	4.0	1.3819	3.9797	6.8415	9.8315	12.8829	15.9660	19.0668	22.1786	25.2975	28.4212
.2	6.0	1.4666	4.1548	7.0190	9.9858	13.0137	16.0776	19.1634	22.2634	25.3728	28.4889
.2	8.0	1.5149	4.2690	7.1524	10.1138	13.1289	16.1795	19.2537	22.3439	25.4452	28.5545
.2	10.0	1.5461	4.3482	7.2540	10.2189	13.2288	16.2713	19.3372	22.4196	25.5141	28.6175
.2	20.0	1.6136	4.5336	7.5209	10.5299	13.5562	16.5981	19.6539	22.7217	25.7999	28.8868
.2	100.0	1.6730	4.7078	7.8018	10.9053	14.0122	17.1209	20.2306	23.3412	26.4525	29.5644
.2	INFY	1.6887	4.7544	7.8794	11.0137	14.1513	17.2903	20.4301	23.5704	26.7110	29.8518
.3	.3	.7558	3.3217	6.3772	9.4880	12.6139	15.7461	18.8813	22.0184	25.1566	28.2955
.3	.4	.8118	3.3498	6.3926	9.4984	12.6218	15.7524	18.8866	22.0229	25.1606	28.2991
.3	.5	.8612	3.3772	6.4078	9.5089	12.6297	15.7587	18.8919	22.0275	25.1645	28.3026
.3	1.0	1.0438	3.5049	6.4825	9.5604	12.6688	15.7902	18.9182	22.0501	25.1843	28.3202
.3	2.0	1.2465	3.7159	6.6218	9.6600	12.7456	15.8524	18.9704	22.0950	25.2238	28.3553
.3	4.0	1.4331	4.0016	6.8551	9.8413	12.8905	15.9721	19.0720	22.1831	25.3014	28.4247
.3	6.0	1.5181	4.1761	7.0322	9.9954	13.0212	16.0837	19.1686	22.2678	25.3767	28.4924
.3	8.0	1.5666	4.2900	7.1654	10.1232	13.1362	16.1855	19.2588	22.3483	25.4491	28.5580
.3	10.0	1.5979	4.3690	7.2668	10.2232	13.2361	16.2773	19.3422	22.4240	25.5180	28.6210
.3	20.0	1.6659	4.5543	7.5335	10.5391	13.5634	16.6039	19.6588	22.7260	25.8037	28.8902
.3	100.0	1.7257	4.7285	7.8144	10.9143	14.0193	17.1267	20.2355	23.3454	26.4562	29.5678
.3	INFY	1.7414	4.7751	7.8920	11.0228	14.1584	17.2961	20.4350	23.5747	26.7148	29.8552
.4	.4	.8657	3.3774	6.4079	9.5089	12.6297	15.7587	18.8919	22.0275	25.1645	28.3026
.4	.5	.9135	3.4044	6.4231	9.5193	12.6376	15.7650	18.8972	22.0320	25.1685	28.3061
.4	1.0	1.0923	3.5304	6.4974	9.5707	12.6766	15.7965	18.9235	22.0546	25.1883	28.3237
.4	2.0	1.2955	3.7393	6.6361	9.6701	12.7533	15.8586	18.9756	22.0995	25.2277	28.3588
.4	4.0	1.4803	4.0232	6.8687	9.8511	12.8981	15.9783	19.0772	22.1875	25.3053	28.4282
.4	6.0	1.5657	4.1970	7.0453	10.0049	13.0286	16.0898	19.1737	22.2723	25.3806	28.4959
.4	8.0	1.6145	4.3107	7.1783	10.1326	13.1436	16.1916	19.2639	22.3527	25.4550	28.5614
.4	10.0	1.6460	4.3896	7.2796	10.2375	13.2433	16.2833	19.3473	22.4284	25.5218	28.6244
.4	20.0	1.7145	4.5747	7.5461	10.5481	13.5705	16.6098	19.6638	22.7303	25.8075	28.8936
.4	100.0	1.7747	4.7490	7.8269	10.9234	14.0263	17.1324	20.2404	23.3497	26.4600	29.5711
.4	INFY	1.7906	4.7956	7.9045	11.0318	14.1654	17.3019	20.4399	23.5789	26.7185	29.8585
.5	.5	.9602	3.4310	6.4382	9.5296	12.6454	15.7713	18.9024	22.0365	25.1725	28.3097
.5	1.0	1.1362	3.5555	6.5122	9.5809	12.6844	15.8028	18.9287	22.0591	25.1923	28.3273
.5	2.0	1.3385	3.7623	6.6504	9.6801	12.7610	15.8649	18.9809	22.1040	25.2317	28.3624
.5	4.0	1.5239	4.0445	6.8822	9.8608	12.9056	15.9844	19.0824	22.1920	25.3092	28.4317
.5	6.0	1.6098	4.2177	7.0584	10.0144	13.0361	16.0958	19.1788	22.2767	25.3845	28.4994
.5	8.0	1.6590	4.3311	7.1912	10.1419	13.1509	16.1976	19.2690	22.3571	25.4569	28.5649

TABLE I (Continued)

N_{B1}	N_{B2}	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_9	ϵ_{10}
.5	10.0	1.6908	4.4099	7.2924	10.2467	13.2506	16.2892	19.3523	22.4328	25.5257	28.6279
.5	20.0	1.7599	4.5949	7.5587	10.5572	13.5776	16.6156	19.6687	22.7346	25.8113	28.8970
.5	100.0	1.8206	4.7692	7.8394	10.9324	14.0334	17.1382	20.2453	23.3539	26.4637	29.5745
.5	INFY	1.8366	4.8158	7.9171	11.0408	14.1724	17.3076	20.4448	23.5831	26.7222	29.8619
1.0	1.0	1.3065	3.6732	6.5846	9.6317	12.7232	15.8341	18.9550	22.0817	25.2120	28.3449
1.0	2.0	1.5094	3.8712	6.7202	9.7299	12.7993	15.8960	19.0070	22.1265	25.2514	28.3799
1.0	4.0	1.7004	4.1458	6.9485	9.9090	12.9432	16.0151	19.1082	22.2143	25.3288	28.4492
1.0	6.0	1.7902	4.3164	7.1227	10.0616	13.0730	16.1261	19.2044	22.2988	25.4040	28.5168
1.0	8.0	1.8419	4.4288	7.2544	10.1883	13.1873	16.2275	19.2944	22.3791	25.4762	28.5822
1.0	10.0	1.8753	4.5073	7.3550	10.2926	13.2867	16.3189	19.3775	22.4546	25.5450	28.6451
1.0	20.0	1.9480	4.6919	7.6204	10.6022	13.6129	16.6447	19.6934	22.7561	25.8303	28.9140
1.0	100.0	2.0119	4.8664	7.9010	10.9771	14.0684	17.1669	20.2697	23.3751	26.4824	29.5912
1.0	INFY	2.0288	4.9132	7.9787	11.0855	14.2074	17.3364	20.4692	23.6043	26.7409	29.8786
2.0	2.0	1.7207	4.0575	6.8512	9.8264	12.8746	15.9573	19.0587	22.1711	25.2906	28.4149
2.0	4.0	1.9262	4.3218	7.0734	10.0025	13.0170	16.0756	19.1594	22.2586	25.3678	28.4840
2.0	6.0	2.0246	4.4892	7.2443	10.1530	13.1455	16.1859	19.2551	22.3426	25.4428	28.5514
2.0	8.0	2.0816	4.6006	7.3741	10.2784	13.2590	16.2867	19.3447	22.4228	25.5146	28.6167
2.0	10.0	2.1186	4.6787	7.4736	10.3818	13.3576	16.3776	19.4275	22.4981	25.5834	28.6795
2.0	20.0	2.1993	4.8639	7.7378	10.6897	13.6823	16.7022	19.7424	22.7987	25.8661	28.9479
2.0	100.0	2.2703	5.0396	8.0184	11.0642	14.1373	17.2238	20.3180	23.4171	26.5196	29.6245
2.0	INFY	2.2889	5.0870	8.0962	11.1727	14.2764	17.3932	20.5175	23.6463	26.7781	29.9119
4.0	4.0	2.1537	4.5779	7.2872	10.1740	13.1567	16.1923	19.2591	22.3454	25.4446	28.5527
4.0	6.0	2.2653	4.7439	7.4535	10.3211	13.2831	16.3013	19.3540	22.4290	25.5191	28.6198
4.0	8.0	2.3306	4.8559	7.5810	10.4442	13.3949	16.4010	19.4428	22.5085	25.5908	28.6849
4.0	10.0	2.3731	4.9351	7.6793	10.5461	13.4924	16.4910	19.5250	22.5833	25.6590	28.7474
4.0	20.0	2.4664	5.1244	7.9425	10.8516	13.8145	16.8133	19.8380	22.8825	25.9425	29.0148
4.0	100.0	2.5488	5.3054	8.2246	11.2261	14.2688	17.3339	20.4125	23.4997	26.5929	29.6904
4.0	INFY	2.5704	5.3540	8.3029	11.3348	14.4080	17.5034	20.6120	23.7289	26.8514	29.9778
6.0	6.0	2.3849	4.9113	7.6175	10.4659	13.4079	16.4091	19.4481	22.5121	25.5933	28.6867
6.0	8.0	2.4554	5.0251	7.7440	10.5874	13.5184	16.5078	19.5362	22.5911	25.6646	28.7514
6.0	10.0	2.5015	5.1060	7.8420	10.6884	13.6150	16.5971	19.6178	22.6654	25.7325	28.8137
6.0	20.0	2.6029	5.3005	8.1061	10.9926	13.9352	16.9175	19.9292	22.9633	26.0148	29.0803
6.0	100.0	2.6928	5.4875	8.3910	11.3680	14.3893	17.4375	20.5028	23.5795	26.6643	29.7549
6.0	INFY	2.7165	5.5378	8.4703	11.4773	14.5288	17.6072	20.7024	23.8088	26.9228	30.0423
8.0	8.0	2.5292	5.1409	7.8703	10.7081	13.6280	16.6059	19.6238	22.6697	25.7355	28.8159
8.0	10.0	2.5776	5.2234	7.9685	10.8085	13.7238	16.6945	19.7048	22.7436	25.8031	28.8779
8.0	20.0	2.6844	5.4227	8.2344	11.1124	14.0428	17.0134	20.0148	23.0402	26.0845	29.1437

TABLE E1 (Concluded)

N_{B1}	N_{B2}	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_9	ϵ_{10}
8.0	100.0	2.7794	5.6151	8.5227	11.4894	14.4974	17.5333	20.5879	23.6556	26.7331	29.8175
8.0	INFY	2.8044	5.6669	8.6031	11.5993	14.6374	17.7032	20.7877	23.8851	26.9917	30.1049
10.0	10.0	2.6277	5.3073	8.0671	10.9087	13.8192	16.7827	19.7855	22.8173	25.8704	28.9397
10.0	20.0	2.7383	5.5107	8.3351	11.2129	14.1375	17.1005	20.0944	23.1128	26.1509	29.2047
10.0	100.0	2.8368	5.7075	8.6269	11.5920	14.5930	17.6205	20.6672	23.7276	26.7988	29.8778
10.0	INFY	2.8628	5.7606	8.7083	11.7027	14.7335	17.7908	20.8672	23.9574	27.0576	30.1652
20.0	20.0	2.8577	5.7255	8.6116	11.5211	14.4562	17.4166	20.4005	23.4054	26.4284	29.4669
20.0	100.0	2.9648	5.9354	8.9165	11.9107	14.9190	17.9409	20.9752	24.0205	27.0753	30.1383
20.0	INFY	2.9930	5.9921	9.0018	12.0250	15.0625	18.1136	21.1772	24.2516	27.3352	30.4266
100.0	100.0	3.0800	6.1601	9.2405	12.3212	15.4023	18.4840	21.5663	24.6494	27.7333	30.8180
100.0	INFY	3.1105	6.2211	9.3317	12.4426	15.5537	18.6650	21.7767	24.8888	28.0013	31.1143
INFY	INFY	3.1416	6.2832	9.4248	12.5664	15.7080	18.8496	21.9911	25.1327	28.2743	31.4159

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13. ABSTRACT

Exact solutions for the transient temperature distribution and the stored energy in an infinite plate of finite thickness are presented for the case of different convective environments at each face of the plate. The solution is general and contains numerous limiting cases, including that of steady state. Eigenvalues are given for many combinations of the system Biot numbers for the initial response period. An example is presented to illustrate the application of the solution to the practical problem of a rocket engine diffuser.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
temperature						
1 flat plate -- Heat transfer						
energy						
diffuser						
15.7						

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